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The propagation of shock waves in anisotropic materials☆

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ABSTRACT

The propagation of shock waves in anisotropic materials (aluminium alloys), the state of which obeys a non-linear relation, which generalizes Grüneisen's equation for isotropic materials, is investigated. The concept of total generalized pressure and the pressure corresponding to the thermodynamic response is proposed. A modification of the anisotropic Hill criterion in the case of non-associated plastic flow, for which the yield surface is independent of the generalized hydrostatic stress, is considered.

The first attempts[1] to construct an equation of state for orthotropic materials reduced to modifying the classical expression for the hydrostatic pressure, based on the generalized Hooke's law and represented in the form of the sum of two terms, corresponding the volume strain and the strain deviator. The term corresponding to the volume strain was replaced by Grüneisen's equation of state. One other difference from the classical approach was the method of approximating the Hugoniot pressure. However, it was shown in Ref. 2 that this approach leads to breakdown of the invariance of the definition of pressure, corresponding to the thermodynamic response (i.e., the pressure, described by the equation of state). It was shown later,[3,4] on the basis of experiments on the shock loading of zirconium, that a change in the Hugoniot elastic limit as a function of the orientation of the fibre corresponds to a change in the yield point in the case of quasistatic loading of the fibre. Hence, the equation of state[2] and the model of elastoplastic or elastoviscoplastic processes[1,2] serve as the fundamental description of the anisotropic behaviour of materials.

A mathematically correct equation of state for anisotropic materials was constructed in Ref. 2 and two models of anisotropic associated plasticity were considered. By comparing the results of numerical and natural experiments the question was raised regarding the need to construct and use non-associated plasticity together with the anisotropic equation of state,[2] which is the problem considered in this paper. Numerical calculations of shock-wave phenomena in an anisotropic material (aluminium alloy) using a model of associated plasticity and the classical isotropic description of Grüneisen's equation of state, exist,[5] but, as was noted earlier,[1] this approach is mathematically and physically incorrect.

1. Constitutive relations for anisotropic solids under shock loading

The basis of the constitutive relations for anisotropic solids under shock loading is the concept of generalized decomposition of the stress tensor into a generalized pressure and a generalized stress deviator.[2,6]

Decomposition of the stress tensor. We will define the components of the total pressure tensor $\tilde{P}$ as the product of the scalar by the components of the tensor $\alpha = (\alpha_{ij})$

$$\tilde{P}_{ij} = -p_* \alpha_{ij} \tag{1.1}$$

where p_* is the value of the total generalized pressure. We will require that the stress deviator should be independent of the generalized pressure; their convolution must be equated to zero:

$$\tilde{P} : \tilde{S} = 0, \; \alpha_{ij}\tilde{S}_{ij} = 0, \; \sigma_{ij} = -p_* \alpha_{ij} + \tilde{S}_{ij} \tag{1.2}$$

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Using relations (1.1) and (1.2), we obtain the following expressions for the scalar p_* and the components of the generalized stress deviator $\tilde{S}_{ij}$:

$$p_* = -\frac{1}{\|\alpha\|^2}\sigma_{ij}\alpha_{ij}, \quad \tilde{S}_{ij} = \sigma_{ij} - \alpha_{ij}\frac{1}{\|\alpha\|^2}\sigma_{kl}\alpha_{kl} \tag{1.3}$$

where $\|\alpha\|^2 = \alpha_{ij}\alpha_{ij} = \alpha_{11}^2 + \alpha_{22}^2 + \alpha_{33}^2$ is the square of the Frobenius norm for a diagonal tensor. An algorithm for calculating the components α_{ij} is known.[2,6] These components and the generalized tension-compression modulus K_C are defined as follows:

$$\alpha_{kk} = \left(\sum_i C_{ki}\right)3\bar{K}_C, \quad k = 1,2,3; \quad \|\alpha\|^2 = 3 \tag{1.4}$$

$$K_C = \frac{1}{9\bar{K}_C} = \frac{1}{3\sqrt{3}}\sqrt{\left(\sum_i C_{1i}\right)^2 + \left(\sum_i C_{2i}\right)^2 + \left(\sum_i C_{3i}\right)^2} \tag{1.5}$$

where C_{ij} are the components of the symmetric elastic-constant tensor. In the case of isotropy, the generalized pressure reduces to the classical hydrostatic pressure. The parameters α_{ij} and K_C describe the fundamental properties of an anisotropic material.

Convolution of the stress tensor with an arbitrary tensor $\beta = (\beta_{ij})$ within framework of the generalized Hooke's law gives

$$\beta_{ij}\sigma_{ij} = \beta_{ij}\alpha_{ij}K_C\varepsilon + \beta_{ij}C_{ijkl}e_{kl} = -\beta_{ij}\alpha_{ij}p + \beta_{ij}C_{ijkl}e_{kl} \tag{1.6}$$

where e_{kl} are the components of the elastic strain deviator, for which we require.

$$\beta_{ij}C_{ijkl}e_{kl} = 0 \tag{1.7}$$

We then obtain from relations (1.6) and (1.7)

$$p = -\sigma_{ij}\beta_{ij}/(\alpha_{kl}\beta_{kl}) \tag{1.8}$$

The parameter p depends directly on the spherical strain tensor and is an invariant quantity, since it is obtained as a result of tensor operations. In problems of the behaviour of a material under shock load the physical quantity p is defined by Grüneisen's equation of state. In this case also, the discussions presented above enable the behaviour of a material under high pressure to be correctly described.

An algorithm for calculating the components β_{ij} was constructed in Ref. 2 on the basis of relation (1.7); these components have the form

$$\beta_{kk} = \left(\sum_i J_{ki}\right)3K_S, \quad k = 1,2,3; \quad \|\beta\|^2 = 3 \tag{1.9}$$

$$\frac{1}{K_S} = \sqrt{3}\sqrt{\left(\sum_i J_{1i}\right)^2 + \left(\sum_i J_{2i}\right)^2 + \left(\sum_i J_{3i}\right)^2} \tag{1.10}$$

where J_{ij} are the components of the symmetrical pliability tensor.

After determining the parameter p in terms of the second-order tensors α and β and the stress tensor, we can write the relation between the total generalized pressure p_* and the pressure p

$$\beta_{ij}\alpha_{ij}p = -\beta_{ij}\sigma_{ij} = -\beta_{ij}(-\alpha_{ij}p_* + \tilde{S}_{ij}) \tag{1.11}$$

whence it follows that

$$p_* = p + \beta_{ij}\tilde{S}_{ij}/(\alpha_{kl}\beta_{kl}) \tag{1.12}$$

i.e., the stress tensor is represented in the form of the sum of a generalized spherical tensor and a generalized deviator. The use of the fundamental tensors α and β enabled us to determine the total generalized pressure and the pressure corresponding to volume deformations. The expressions obtained reduce to the classical relations for an isotropic solid.

The first K_C and second K_S generalized bulk tension-compression moduli were determined. In the isotropic case, both moduli reduce to the classical form of the bulk modulus.[2]

Grüneisen's equation of state (the isotropic case). The equation of state for isotropic solids defines the pressure as a function of the density ρ (or the specific volume ν and the specific internal energy e. The following Grüneisen's equation of state is widely used:[7,8]

$$p = f(\rho, e) = P_r(\nu) + \frac{\Gamma(\nu)}{\nu}(e - e_r(\nu)); \quad \Gamma(\nu) = \nu\left(\frac{\partial p}{\partial e}\right)_\nu \tag{1.13}$$

where $\Gamma(\nu)$ is Grüneisen's parameter[7–11]. Note that different definitions of Grüneisen's parameter exist, including the widely used definition, given by Mie-Grüneisen[9] in terms of the ratio of the thermal pressure to the specific thermal energy. Traditionally, the parameter Γ is fixed: $\Gamma = \Gamma_0$; the assumption $\nu\Gamma_0 = \nu_0\Gamma$ is another version. The functions $P_r(\nu)$ and $e_r(\nu)$ are derived by processing the calibration curve, for which one can take the Hugoniot shock adiabatic curve, the standard adiabatic curve, passing through the initial point of the state (p_0,

ν_0), the isotherm at 0 K, the isobar $p=0$, the curve $e=0$, and a composition of curves covering the working range of the parameter ν. A form of Grüneisen's equation of state is widely used for an isotropic deformable solid with a Hugoniot calibration curve[1,2,5,7–11]

$$p = f(\rho, e) = P_H\left(1 - \frac{\Gamma}{2}\mu\right) + \rho\Gamma(e - e_0); \quad \mu = \frac{\rho}{\rho_0} - 1 \tag{1.14}$$

where P_H is the Hugoniot pressure, μ is the relative change in volume, and e_0 is the initial specific internal energy. Henceforth we will assume that the initial specific internal energy is equal to zero.[1,2,5,7–11]

The Rankine Hugoniot condition for discontinuous solutions can be found from the relation between any two of the variables ρ, p, e, u_p and U.[8] The Hugoniot pressure and the shock wave velocity U are functions of the particle velocities u_p,[2] and are related as follows[7,10,11]:

$$U = c + S_1 u_p + S_2(u_p/U)u_p + S_3(u_p/U)^2 u_p \tag{1.15}$$

where c is the bulk velocity of sound. Grüneisen's parameter is[7,10,11]

$$\Gamma = (\gamma_0 + a\mu)/(1+\mu) \tag{1.16}$$

where γ_0 is the initial value of the Grüneisen's parameter and a is the bulk correction factor to the value of γ_0. As a cosequence, Grüneisen's equation for a cubic dependence of U on u_p can be represented as follows:

$$p = \begin{cases} \dfrac{\rho_0 c^2\mu\left[1 + \left(1 - \dfrac{\Gamma}{2}\right)\mu - \dfrac{\Gamma}{2}\mu^2\right]}{\left[1 - (S_1 - 1)\mu - S_2\dfrac{\mu^2}{\mu+1} - S_3\dfrac{\mu^3}{(\mu+1)^2}\right]^2} + (1+\mu)\Gamma E, & \mu > 0 \\ \rho_0 c^2\mu + (1+\mu)\Gamma E, & \mu < 0 \end{cases} \tag{1.17}$$

where E is the internal energy per unit initial density ($\rho_0 E = e$).[7,10,11] The parameters c, S_1, S_2, S_3, γ_0 and a reflect the material properties determining the equations of state.

In the case of an isotropic material the hydrostatic stress induces a bulk change in the material, whereas the stress deviator produces a change in shape. To preserve this property in anisotropic materials it is necessary to generalize the definition of the idea of a "hydrostatic stress", since the loading of an anisotropic solid by a hydrostatic pressure, defined in the traditional way, leads to a change in shape, which expresses the intrinsic contradiction of the model.

The equation of state for an anisotropic solid. Suppose the total "hydrostatic pressure" is expressed by relation (1.2). To describe the behaviour of the material at high pressures a corresponding equation of state is necessary, and it is proposed to write it in the form of (1.17). The expression for the total generalized hydrostatic pressure at high levels takes the form

$$p_* = p_{\text{EOS}} + \beta_{ij}\tilde{S}_{ij}/(\alpha_{kl}\beta_{kl}) \tag{1.18}$$

The pressure p_{EOS} is described by Eq. (1.17). Relation (1.18) also correctly describes the behaviour of materials for small strains.

We will define the bulk velocity of sound as

$$c = \sqrt{K_C/\rho_0} = c_0 \tag{1.9}$$

The first generalized tension-compression bulk modulus K_C is given by expression (1.5).

A mathematically correct model of incompressible plastic flow for anisotropic materials. A mathematically and thermodynamically correct model of incompressible anisotropic flow is based on the following assumptions[2,6]

$$1)\ \hat{F}(\tilde{S}_{ij}, p_*) = Y(\bar{\varepsilon}_p),\ 2)\ D_{ij} = D_{ij}^e + D_{ij}^p,\ D_{kk}^p = 0,\ 3)\ D_{ij}^p = \dot{\lambda}\partial g(\sigma_{ij}, \gamma_k)/\partial\sigma_{ij}$$

The function $\hat{F}(\tilde{S}_{ij}, p_*)$ describes the yield surface, $g = g(\sigma_{ij}, \gamma_k)$ is the plastic potential, D_{ij} are the components of the total strain rate tensor, D_{ij}^e are the components of the elastic strain rate tensor, D_{ij}^p are the components of the plastic strain rate tensor, γ_k is the set of internal parameters (for example, the strengthening parameters), σ_{ij} are the components of the stress tensor, and $\tilde{S}_{ij}$ are the components of the generalized stress deviator.

The majority of mathematical models of anisotropic plasticity are based on the following fundamental assumptions[12]: in the plastic state the medium is incompressible (i.e., $D_{kk}^p = 0$) and the yield surface is independent of the hydrostatic stress. It was shown in Ref. 6 that in these assumptions for the general case of an anisotropic solid there is a stress state corresponding solely to (elastic) volume strains, but which do not satisfy the plasticity criterion. Hence, the models are intrinsically contradictory.

Using Assumption 1, we can construct an intrinsically non-contradictory model of anisotropic plasticity.[2,6] Assumptions 2 and 3 are traditional for the theory of plasticity. We will take as the yield surface a modification of Hill's criterion, in which the effect of the generalized hydrostatic stress is ignored[2,6]:

$$F\left(\tilde{S}_{yy} - \tilde{S}_{zz}\right)^2 + G\left(\tilde{S}_{zz} - \tilde{S}_{xx}\right)^2 + H\left(\tilde{S}_{xx} - \tilde{S}_{yy}\right)^2 + 2N\tilde{S}_{xy}^2 + 2L\tilde{S}_{yz}^2 + 2M\tilde{S}_{xz}^2 = Y^2(\bar{\varepsilon}_P) \tag{1.20}$$

where F, G, H, N, L and M are parameters of the medium. To describe the plastic potential $g = g(\sigma_{ij}, \gamma_k)$ we will use a Hill function[10–12] of the form

$$\frac{1}{R+1}\left[\bar{F}(\sigma_{yy} - \sigma_{zz})^2 + \bar{G}(\sigma_{zz} - \sigma_{xx})^2 + \bar{H}(\sigma_{xx} - \sigma_{yy})^2 + D\right] - \sigma_p^2 \tag{1.21}$$

where

$$D = 2\bar{N}\sigma_{xy}^2 + 2\bar{L}\sigma_{yz}^2 + 2\bar{M}\sigma_{xz}^2$$

and $\bar{F}, \bar{G}, \bar{H}, \bar{N}, \bar{L}, \bar{M}$ are parameters of the medium for the plastic potential, defined in terms of the anisotropy parameters $R, P, Z, Q_{yz}, Q_{yx}, Q_{zx}$ [11]

$$\bar{F} = R/P,\ \bar{G} = R,\ \bar{H} = 1,\ \bar{N} = (Q_{yx} + 1/2)(R + Z) \tag{1.22}$$

$$\bar{M} = (Q_{zx} + 1/2)(R + Z),\quad \bar{L} = (Q_{yz} + 1/2)(Z + 1) \tag{1.23}$$

which, in turn, are established from experiments on uniaxial stretching.[11]

2. Numerical modelling of the propagation of shock waves

We investigated the propagation of shock waves in the anisotropic aluminium alloy AA7010-T6.

Description of the experiments.[5] A perturbation is produced by the collision of a thin plate-striker (6082-T6 dural) and a thin plate-target (AA7010-T6 aluminium alloy). The plane wave front of the shock wave propagates in the plate-target. The shock-wave pulse is recorded by a piezometric sensor, situated between the target and a plate of perspex of width 12 mm, fixed to the target.

In a series of experiments the plate-striker, 2.5 mm thick, hits the plate-target with different velocities in the range from 234 to 895 m/s, producing stresses in the range from 2.7 GPa to 7.2 GPa. The properties of AA7010-T6 are characterized by the following values[5]: initial density $\rho_0 = 2810\ \text{kg/m}^3$, Young's modulus in the longitudinal direction (direction 1) $E_1 = 70.6$ GPa, Young's modulus in the transverse direction (direction 2) $E_2 = 71.1$ GPa, Young's modulus in a direction perpendicular to directions 1 and 2 (direction 3) $E_3 = 70.6$ GPa, Poisson's ratios $\nu_{12} = 0.342$, $\nu_{31} = 0.342$ and $\nu_{23} = 0.342$, and the shear moduli in directions 12, 31 and 23 are $G_{12} = 26.3$ GPa, and $G_{31} = G_{23} = 26.5$ GPa. To describe the elastic behaviour of AA7010-T6 orthotropic material nine constants are sufficient, as given previously in Refs 2, 5, 10 and 11. Note that Poisson's ratios ν_{ij} for an anisotropic material are taken in Refs 10 and 11 as the ratio of the strain ε_j in the j direction to the strain ε_i in the i direction, taken with the opposite sign. The material properties of the plates of 6082-T6 and perspex were given in Refs 2 and 7.

A verification of the constitntive relations. Numerical modelling of the collision of the plates was carried out with a plate-striker 2.5 mm thick, a plate-target 5 mm thick and a plate of perspex 5 mm thick. The remaining dimensions of the plates were assumed to be much greater. A more detailed description of the numerical experiment can be found in Ref. 2. Calculations were carried out by the finite element method, the number of elements being chosen so that all the elastoplastic properties of the shock wave could be calculated with sufficient accuracy.[2] The collision of the plates was modelled for shock velocities $V = 200$, 504 and 700 m/s.

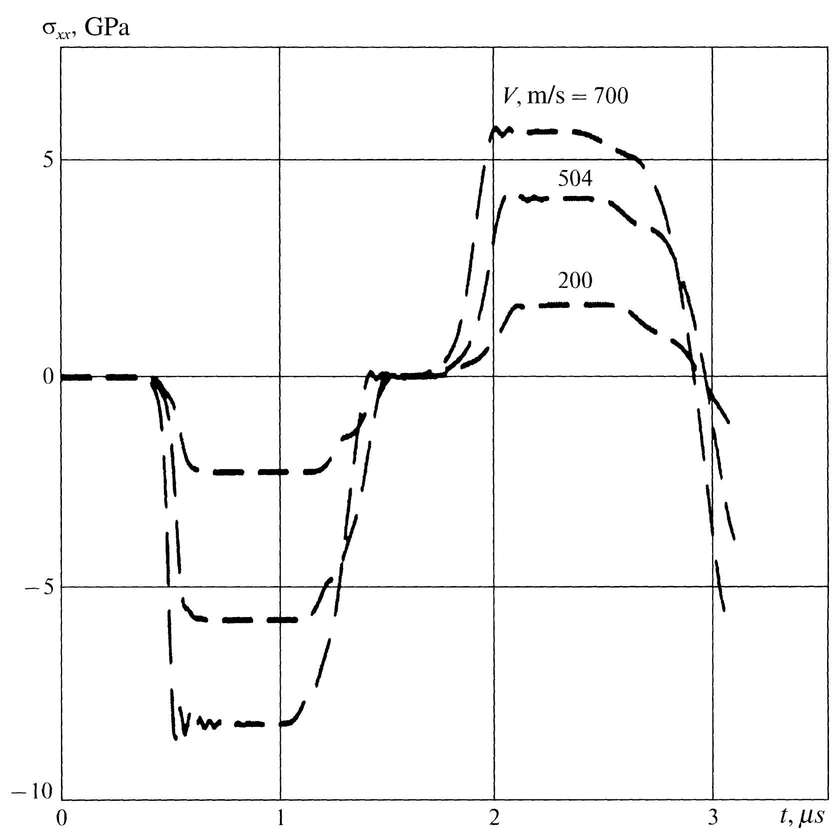

Fig. 1.

When generalizing the constitutive relations to the case if anisotropic materials, the main requirement was the possibility of reducing the generalized model to the classical version for isotropic materials. This reduction is observed at a theoretical level. We will confirm this numerically using the example of the problem of the collision of thin isotropic plates (a plate-striker of aluminium and a plate-target of solid copper; the mechanical properties of the materials correspond to those used previously in Ref. 7). The parameters of the target were as follows: density $\rho_0 = 8930\,\mathrm{kg/m^3}$, shear modulus $G_0 = 47.7$ GPa, and yield point $Y_0 = 120$ MPa. We assumed ideally elastoplastic behaviour of plate-target. The parameters of the Grüneisen's equation of state (1.16), (1.17) for the isotropic plate-target are as follows:

$$c = 5328\ \mathrm{m/s}, \quad S_1 = 1.338, \quad S_2 = S_3 = 0, \quad \gamma_0 = 2.0, \quad a = 0.48$$

In Fig. 1 we show the dynamics of the longitudinal component of the stress σ_{xx} in the middle of the plate-target for different impact velocities V. We can conclude that the Hugoniot elastic limit σ_{HEL} and the maximum value of $|S_{xx}|$ agree well with the values

$$\sigma^T_{\mathrm{HEL}} = \left(\frac{1-\nu}{1-2\nu}\right) Y_0 = 253.8\ \mathrm{MPa}, \qquad \max|S_{xx}|^t = \frac{2}{3} Y_0 = 80\ \mathrm{MPa}$$

derived theoretically using the Mises isotropic criterion. The Rankine-Hugoniot condition for the discontinuous solutions $U = c + Su_p$ enables us to determine the shock wave velocity and to approximate the amplitude of the stress behind the loading shock wave:

$$\sigma_{xx} = P + S_{xx} = \rho_0 U u_p + \frac{2}{3} Y_0 \tag{2.1}$$

The numerical values of $\sigma_{xx} = 2.234$, 5.733 and 8.145 GPa, obtained by modelling, agree with the analytical approximations $\sigma_{xx} = 2.239$, 5.735 and 8.152 GPa for particle velocities $u_p = 60.1$ m/s (an impact velocity $V = 200$ m/s), 152.8 m/s (an impact velocity $V = 504$ m/s), and 213.9 m/s (an impact velocity $V = 700$ m/s) respectively.

Modelling of the shock waves propagation in an anisotropic aluminium alloy. An experimental investigation of the effect of the anisotropy of the material on the shock wave propagation was carried out as follows: a striker (made of 6082-T6 dural) was fired at a target (AA7010-T6 alloy) at velocities of 450 m/s and 895 m/s. The parameters of Grüneisen's equation of state (1.16), (1.17) for the plate-striker were taken from Ref. 7. The parameters of the generalized equation of state (1.17)–(1.19) for AA7010-T6 alloy had the following values.

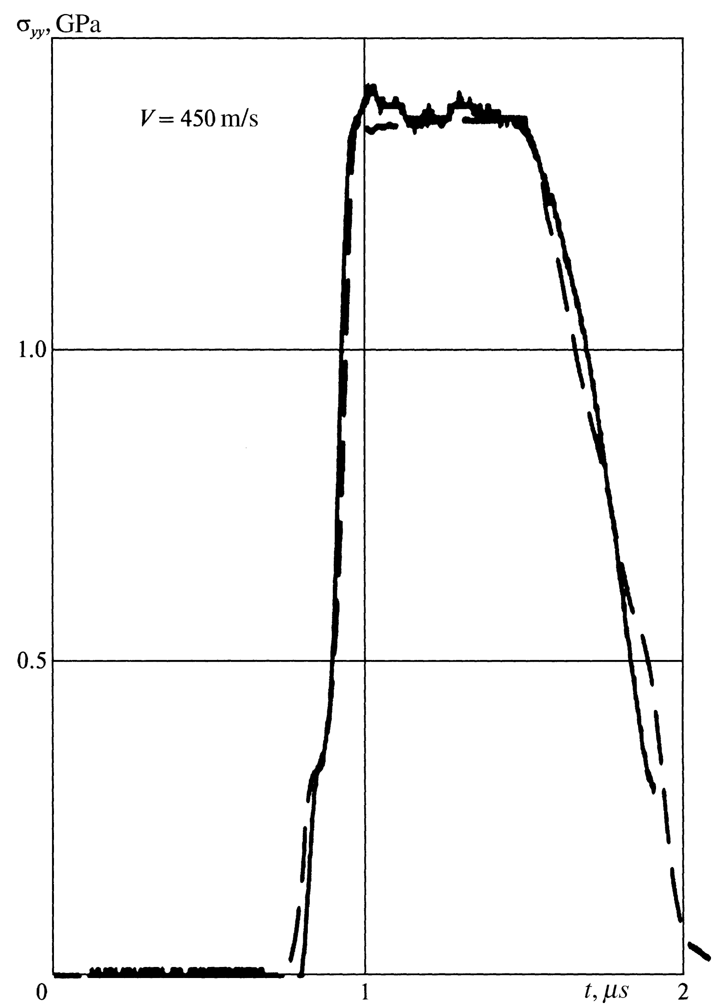

Fig. 2.

$$c_0 = \sqrt{K_C/\rho_0} = 5154 \text{ m/s}, \quad K_C = 74.65 \text{ GPa}, \quad S_1 = 1.4$$
$$S_2 = S_3 = 0, \quad \gamma_0 = 2.0, \quad a = 0.48$$

They were found by approximating the actual values obtained from experiments in which the effect of plastic anisotropy can be neglected. The model of anisotropic plasticity was described by relations (1.20) and (1.21), containing 12 unknown parameters of the yield surface and the plastic potential. In a collision with thin plates under conditions of uniaxial strain it is only necessary to know the parameters F, G and H for the yield surface; they can be found experiments to establish the Hugoniot elastic limit σ_{HEL}. The situation is similar for the plastic potential: it is necessary to establish the values of $\bar{F}$, $\bar{G}$, $\bar{H}$ from an independent experiment.[13,14] In the calculations, we used the following values for the yield surface,

$$F = 0.6898, \quad G = 0.2884, \quad H = 0.6821, \quad Y = 500 \text{ MPa}$$

and for the plastic potential

$$\bar{F} = 0.6954, \quad \bar{G} = 0.5, \quad \bar{H} = 0.5, \quad R = 1, \quad P = 0.719, \quad \sigma_p = 564 \text{ MPa}$$

(in the longitudinal direction, they were taken from Ref. 5).

In Figs. 2 and 3 we compare the results of the numerical calculation of the stress σ_{yy} in the transverse direction with the experimental data for the impact velocity of 450 m/s and the numerical calculation of the stress σ_{xx} in a longitudinal direction with the experimental data for the impact velocity of 895 m/s. The stresses are shown with the opposite sign for convenience in comparing them with the experimental data (the continuous curves). A similar comparison for an impact velocity of 450 m/s in the case of stresses σ_{xx} and for an impact velocity of 895 m/s in the case of stresses σ_{yy} also shows that the experimental values agree well with the calculated values.

A comparison of the results of the calculation with experimental data shows that the experimental values for the Hugoniot elastic limit of 0.39 GPa and 0.33 GPa in the longitudinal and transverse directions (Figs. 3 and 2) agree well with the calculated values of 0.395 GPa and 0.333 GPa. The results obtained also agree with those presented in Ref. 2. The plastic wave velocity also agrees well with the experimental value; this justifies the choice of the plastic potential in the form (1.21). The width of the pulse, determined by the loading and unloading

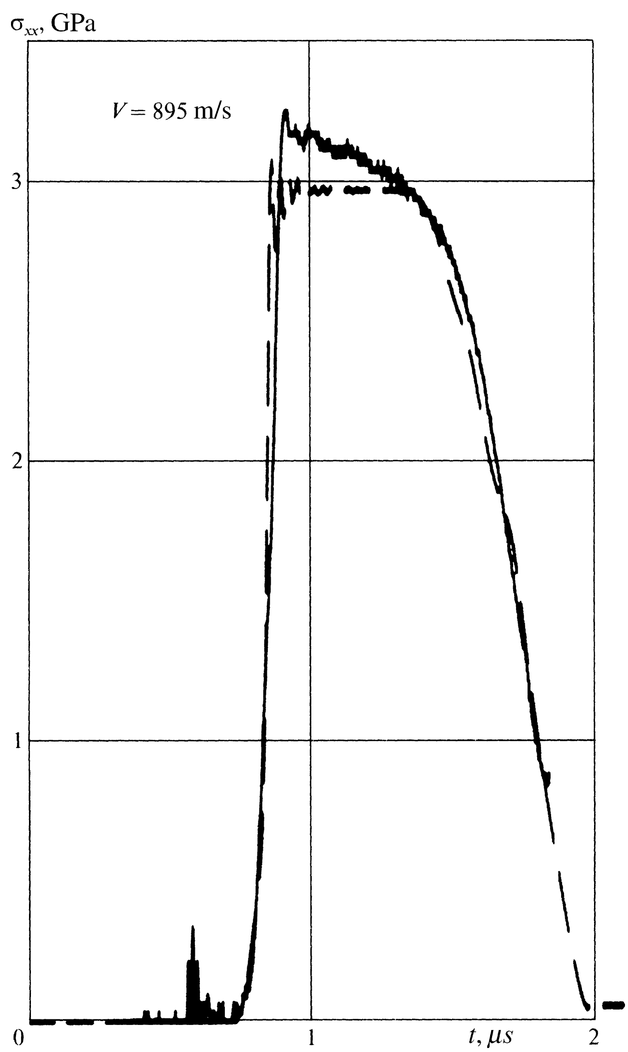

Fig. 3.

waves, also agrees well with that obtained in the experiment. It is worth noting that the stress level behind the loading wave does not agree with the experimental data for the impact velocity of 895 m/s; hence the need arises to take into account the effect of the strain rates and of the generalized stress in the constitutive relations.

3. Conclusions

We have constructed a non-contradictory model of the equation of state of anisotropic materials, suitable for investigating the propagation of shock waves. On the basis of the concept of a generalized decomposition of the stress tensor, we have constructed two fundamental tensors, describing the anisotropy of the material. We have constructed an equation of state for anisotropic solids, which is a generalization of the classic Grüneisen equation for isotropic solids. In the formulation of the model of anisotropic incompressible plasticity, the yield surface is independent of the generalized hydrostatic stress. We have proposed a modification of Hill's criterion, ignoring the generalized hydrostatic stress. We have verified the constitutive relations by comparing the calculated values with the analytical approximations for an isotropic material. By numerical modelling of the propagation of elastoplastic shock waves in an anisotropic alloy (AA7010-T6 aluminium) we have established that the difference in the Hugoniot elastic limits, determined from the results of the calculation, from experimental data does not exceed experimental errors. The calculated profile of the loading and unloading waves agrees with that obtained experimentally. Hence, we have proved the correctness of taking plastic anisotropy into account in the model and the choice of the generalized equation of state.

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